Generalized Make and Use Framework for Allocation in Life Cycle Assessment

Sangwon Suh, Bo Weidema, Jannick Hoejrup Schmidt,
and Reinout Heijungs

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Summary

Allocation in life cycle inventory (LCI) analysis is one of the long-standing methodological issues in life cycle assessment (LCA). Discussion on allocation among LCA researchers has taken place almost in complete isolation from the series of closely related discussions from the 1960s in the field of input–output economics, regarding the supply and use framework. This article aims at developing a coherent mathematical framework for allocation in LCA by connecting the parallel developments of the LCA and the input–output communities. In doing so, the article shows that the partitioning method in LCA is equivalent to the industry-technology model in input–output economics, and system expansion in LCA is equivalent to the by-product-technology model in input–output output economics. Furthermore, we argue that the commodity-technology model and the by-product-technology model, which have been considered as two different models in input–output economics for more than 40 years, are essentially equivalent when it comes to practical applications. It is shown that the matrix-based approach used for system expansion successfully solves the endless regression problem that has been raised in LCA literature. A numerical example is introduced to demonstrate the use of allocation models. The relationship of these approaches with consequential and attributional LCA models is also discussed.

Address correspondence to:
Sangwon Suh
Bren School of Environmental
Science and Management
University of California
Santa Barbara, CA 93106-5131
suh@bren.ucsb.edu

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Introduction

Allocation in life cycle inventory (LCI) analysis is one of the long-standing methodological issues in life cycle assessment (LCA). *Allocation* here refers to the procedure of partitioning inputs and outputs of a multiproduct process or a product system over its multiple products (ISO 2006). Multiproduct processes may occur either as (1) a multiple-output process, whereby a process supplies more than one product; (2) recycling, whereby a process converts at least one waste input into at least one product output; or (3) a multiple-input process, whereby a service, such as transport or waste treatment, is provided simultaneously to several product systems (Lindfors et al. 1995).

Allocation was brought to the attention of the LCA community during its early development era (see, e.g., Heijungs et al. 1992; Huppes and Schneider 1994; Lindfors et al. 1995; Azapagic and Clift 1999). By the late 1990s, international standards were published that included specifications for allocation (ISO 1998), and a number of Ph.D. theses were published that focused on allocation aspects (see, e.g., Azapagic 1996; Frischknecht 1998; Ekvall 1999). In the course of the concept's development in the 1990s, the LCA community recognized two major classes of allocation methods: the partitioning method, and system expansion (Frischknecht 2000; Ekvall and Finnveden 2001; Guinée et al. 2002).[1] In the partitioning method, inputs and outputs of a process are assigned to its multiple products according to a ratio based on the economic value of the products (Huppes 1994), internal cost accounting principles (Frischknecht 1998), or any other factor that is considered to represent the relative share of responsibility. The partitioning ratio in this case should reflect a physical or economic causality between the inputs and outputs and the multiple products of the process in question. In system expansion, coproducts substitute for equivalent products produced elsewhere. The example illustrated in ISO 14041 (ISO 1998, 20) is a waste incineration service that also produces energy. To isolate the waste treatment service, one must subtract the inputs and outputs required to produce equivalent amount of energy elsewhere, which are avoided by the energy produced from the incineration process, from the system in question. System expansion considers the market mechanism under which a substitution between products takes place in reality (ISO 2000,31–32). Ekvall (2000) elaborated on the use of market mechanisms in modeling the substitution. Weidema (2001, 2003) showed how system expansion can be used in various allocation situations.

Choosing an allocation method involves making assumptions on causal mechanisms. Therefore, researchers have also made efforts to refine the decision-making aspects when applying allocation methods (Ekvall and Tillman 1997; Tillman 2000; Guinée et al. 2004). Heijungs and Frischknecht (1998) and Heijungs and Suh (2002) discussed the mathematical formulation of allocation using the matrix notation introduced by Heijungs (1994).

The discussion on allocation among LCA researchers has taken place almost in complete isolation from the series of closely related discussions from the 1960s in the field of input—output economics regarding the supply and use framework (see, e.g., Stone 1961; Van Rijckeghem 1967; UN 1968; Ten Raa et al. 1984; Ten Raa 1988; Kop Jansen and Ten Raa 1990; Steenge 1990; Rainer and Richter 1992; UN 1993; Konijn 1994; Londero 1999). We found only a handful of references in the LCA literature that mention the implementation of the supply and use framework for LCA (see Heijungs 1997, 2001; Heijungs and Suh 2002; Kagawa and Suh 2009).

The resemblance between LCA and input—output analysis (IOA) and their possible synergy were discussed following the first appearance of the term *IOA* on the LCA scene by Moriguchi and colleagues (1993) and Heijungs (1994). Such discussions were often shaped as either pro—contra discussions (e.g., Lave et al. 1995) or pleas for an amalgam in the form of hybrid analysis (e.g., Suh et al. 2004; Suh & Huppes 2005). The issue of the similarity of allocation methods was hardly discussed (Heijungs [1997, 2001] and Kagawa and Suh [2009] offer exceptions).

In this article, we aim at developing a coherent mathematical framework for allocation in LCA by connecting the parallel developments of the LCA and the input—output communities. In doing so, we show that the partitioning method in LCA is equivalent to the industry-technology model in input—output economics and that system expansion in LCA is equivalent to the by-product-technology model in input—output economics. Furthermore, we argue that the commodity-technology model and the by-product-technology model, which have been considered as two different models in input—output economics for more than 40 years, are essentially equivalent when it comes to practical applications. We show that the matrix-based approach used for system expansion successfully solves the endless regression problem that has been raised in the LCA literature. We discuss the relationship between these approaches and consequential and attributional LCA models.

This article uses matrices to represent LCI problems. Although lay users of LCA might not have encountered matrices when carrying out LCAs using software tools and databases, a matrix method has been widely adopted by LCA software tools and databases since Heijungs's (1994) work (see also the work of Heijungs and Suh [2002] and Suh and Huppes [2005]). Many LCA users, ourselves included, utilize the matrix method in their LCA studies. LCA users who utilize matrix methods, as well as software and database developers, can easily adopt the method presented in this article by following our equations. We do not address the subjective aspects of the particular allocation rules (partitioning versus system expansion, mass based versus economics-based, etc.) but rather provide a framework under which consistent mathematical principles are applied in calculations, once a particular allocation rule is chosen.

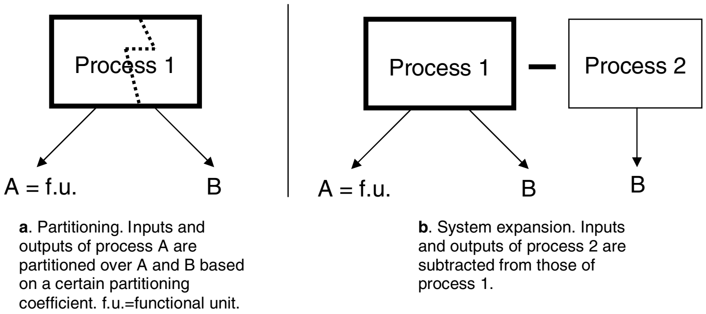
Terms, Definitions, and General Principles

According to ISO standards, the term *products* embraces both goods and services. We use the term synonymously with *commodity* in this article to establish the connection with the input—output and supply—use literature. For the same reason, we use the term *process* synonymously with *industry* and *sector*. For ease of exposition, we restrict the discussion from here on to the multioutput case, which is generalized later in the *Generalization of the Allocation Situation* section. Two types of multiple-output processes are distinguished, namely *subsidiary* production and *joint* production (UN 1993). *Subsidiary production* refers to the type of multiple-output production in which the production lines (unit processes) of each output are fully separable and the product outputs can be independently varied. For example, a facility that produces motorbikes and pianos can be considered a subsidiary production, as the two products do not depend on each other. In principle, one can avoid allocation in this case by collecting data at the unit process (or activity; see Konijn 1994) level. Conversely, a multiple-output production that is not separable and cannot vary its multiple outputs independently is referred to as a *joint production*. Electrolysis of sodium chloride, which produces caustic soda, chlorine, and hydrogen, is an example of a joint production (Guinée et al. 2002). It is notable that, in reality, many multiple-output productions lie in between the perfect subsidiary production and the perfect joint production.

In the case where an additional demand for one of the jointly produced products does not affect the production volume of the process, the product is called a *by-product*. For instance, most of the rare metals are produced as an impurity of mass-produced metals, and some of the rare metals' contribution to the total revenue of the operation is so limited that additional demand placed on those metals does not affect the volume of the operation. Note that the definition of *by-product* is not bound to the type of products per se, as the supply-and-demand relationships may turn a by-product into the main motivation of an operation.

Two generic methods of handling allocation are distinguished—namely *partitioning* and *system expansion* (Guinée et al. 2002; Heijungs and Suh 2002; ISO 2006; see figure 1). In ISO standards, system expansion is recognized as a way to avoid allocation rather than a method of allocation (ISO 1998, 2000). Both procedures reduce systems with multioutput processes to systems with only single-output processes.

In partitioning, the multioutput processes are split into a number of independently varying single-output processes. Each of these single-output processes is associated with only a share



a. Partitioning. Inputs and outputs of process A are partitioned over A and B based on a certain partitioning coefficient. f.u.=functional unit.

b. System expansion. Inputs and outputs of process 2 are subtracted from those of process 1.

Figure 1 Conceptual illustration of partitioning and system expansion.

of the direct burdens (emissions, resource extractions) and the product inputs that lead to indirect burdens (electricity use, materials use, services). The shares are defined by the allocation factors and can be based on mass, energy content, economic value, or any other parameter. They typically add up to 1, and in most cases they are equal for all direct and indirect burdens.

In system expansion, the product system is expanded to include the additional functions related to the coproducts (ISO 1998; see figure 1b). For instance, one can subtract the amount of inputs and outputs attributable to steam production from a cogeneration process by referring to a process that generates only steam. In this way, system expansion seeks to reflect the market reactions to changes in the by-product output (Ekvall 2000; Weidema 2001, 2003). A possible complication in system expansion is when the unit process that produces the coproduct to be subtracted from the multiple-output process under study also produces multiple products. This chain of subtraction continues until it reaches a single-output process. A problem occurs if such a procedure reaches the original process that system expansion started, which is referred to as the *endless regression problem* in system expansion (Weidema 2001). The endless regression problem is at least a conceptual problem in system expansion, although Weidema (2001) pointed out that, in practice, the subsequent outputs become less and less valuable and that the regression therefore can be solved by iteration.

In input–output economics, two types of basic applications are distinguished. One is *impact analysis*, and the other is *imputation study*. Impact analyses are prospective analyses that study the impact of a certain change in final demand on direct and indirect use of factor inputs, whereas imputation studies are retrospective and concerned with assigning the total factor use (possibly including environmental factors) to final demand categories.

In input–output economics, a number of approaches have been developed and used to convert supply–use tables to a square and symmetric input–output table, or so-called A matrix, which represents a unique "recipe" of product inputs required to make a unit of each product. These approaches are referred to as models or assumptions, which are defined and elaborated in the following sections.

Basic Models for Allocation According to the Supply–Use Framework

In this section, we introduce three major models for moving from supply–use tables to analytical tables, plus a combination of these. The basic equations and terms used in this section are drawn from the supply–use frameworks (see, e.g., UN 1968, 1993; Konijn 1994; Konijn and Steenge 1995; Eurostat 2008; Kagawa and Suh 2009). We add to the traditional economic exposition an interpretation in LCA terms. These models have direct relevance to LCA, and we present a numerical example in the *Numerical Examples* section, where these models are used for a simple LCA system.

The three basic models to be discussed here are (1) industry-technology model, (2) commodity-technology model, and (3) by-product-technology model. A mixture of these three models, which is often referred to as a *mixed-technology model*, is discussed as well.

The industry-technology model assumes that each process has its own technology that is not separable over multiple products. In this case, inputs and outputs of the process are distributed over multiple products that are proportional to the chosen allocation key. For example, a refinery process produces various petrochemical products, including gasoline, diesel, kerosene, and asphalt; through this model, inputs and outputs of the refinery process can be distributed over the multiple products on the basis of allocation keys, such as energy contents or economic values of the products. This model is equivalent to the partitioning method in LCA.

The commodity-technology model assumes that each product has its own characteristics in requiring inputs and generating outputs, irrespective of where they are produced. For example, one can separate the inputs and outputs of a motorbike product system from aggregated input—output data from a facility producing both motorbikes and pianos by referring to the input and output data of another facility that produces only pianos. This model is equivalent to the system expansion method in LCA.

The by-product-technology model assumes that production of coproducts is fully dependent on the production of the primary product of a process and treats coproducts as negative inputs to the process. The amount of coproducts is dependent on the production volume of the primary product of the process.

Finally, the mixed technology model uses the above-described three models all together by disaggregating the system into several subsystems and applying the relevant model to each subsystem. In LCA, this can be interpreted as applying different allocation principles to different multi-output processes within the same product system. The framework so far discussed generally covers all coproduct situations using a consistent problem formulation and provides a solution for allocation using the supply and use framework, as we describe below.

Following the notation used in the supply-use framework, let us define a matrix V of processes by products and a matrix U of products by processes, such that an element of each matrix, v_{ij} and u_{ji} shows the amount of product j produced by process i for a year and the amount of product j used by process i during the same period, respectively. Each product can be in any physical or monetary unit, and thus both V and U can be in mixed units.

For convenience, we assume that both V and U are square.[2] Then, the yearly product output, q, and the yearly process output, g, are calculated by

$$q = V'i, \qquad (1)$$

and

$$g = Vi, \qquad (2)$$

respectively, where the prime denotes transposition of a vector or a matrix and i is a summation column vector with the relevant dimension containing only 1s.

Let us define the environmental matrix B, which is an environmental intervention-by-process matrix. Environmental interventions can be any pollutant emitted, land occupied or transformed, or natural resources extracted. Likewise, any other factor inputs, such as wages and profit, can be represented by B. The environmental matrix can take a coefficient form, E, which shows the amount of environmental intervention by each process per unit of its output, such that

$$E = B\hat{g}^{-1}, \qquad (3)$$

where the hat diagonalizes a vector into a square matrix and the superscript inverts a square matrix. Both U and V can take coefficient forms as well (see, e.g., Miller and Blair 1985), although we did not use such coefficient forms to better illustrate elementary operations that form analytical tables.

The Industry-Technology Model

Suppose that the inputs and outputs of a multioutput process are assigned over its multiple products on the basis of the amount of output of each product in a common unit, such as U.S. dollars or megajoules (MJ), regardless of whether they are determining (main) or dependent outputs (by-products). This assumption is referred to as the *industry-technology assumption*, according to the convention of input—output economics, and it agrees with the partitioning method in LCA, using an allocation factor on the basis of the common metric for the outputs (e.g., U.S. dollars or MJ). Under this assumption, the portion of input requirements and environmental intervention needed to produce dependent output of a process are transferred to the processes that produce them by

$$A_I = U\hat{g}^{-1}V\hat{q}^{-1}. \tag{4}$$

and

$$E_I = B\hat{g}^{-1}V\hat{q}^{-1} \tag{5}$$

respectively. Subscript I denotes the industry-technology model. In this case, all of the products of each multiproduct process should be expressed in a common unit through appropriate conversion factors.

According to this method, the total direct and indirect environmental intervention, m_I, or LCI to produce a certain functional unit, k, is calculated by

$$m_I = E_I(I - A_I)^{-1}k \tag{6}$$

where k is a vector of products, in which all entries are zero, except the entry for the product providing the functional unit of the study.

Commodity-Technology Model

Suppose that each product has the same number of input requirements and amount of environmental intervention generation in producing one unit of a product, regardless of the industry in which it is produced. This assumption is known as the *commodity-technology assumption* in the field of input—output economics, and it corresponds to a system expansion method in LCA, with the provision that the "avoided processes" were already included in the technological network that defines the supply and use tables. Under this assumption, the input requirements and environmental interventions needed to produce the nonprimary outputs of a process in a process that produces these inputs as a primary output are subtracted from the process with nonprimary outputs, through the application of

$$A_C = UV'^{-1} \tag{7}$$

and

$$E_C = BV'^{-1}, \tag{8}$$

respectively. Subscript C denotes the commodity-technology model. According to this method, the total direct and indirect environmental intervention, m_c, or LCI to produce a certain functional unit, k, is calculated by

$$m_C = E_C(I - A_C)^{-1}k \tag{9}$$

Note that the commodity-technology model elegantly solves the aforementioned endless regression problem. Without using matrix algebra, one can also use an iterative method or an infinite geometric progression, similar to the methods discussed by Suh and Huppes (2005).

By-product-Technology Model

Suppose that the production of a nonprimary product by each process is fully dependent on the production of primary product. In other words, demand on the nonprimary products of a process does not directly affect the production schedule of the process, but produced nonprimary products can be seen as consumed within the process. This assumption is referred to as the *by-product-technology assumption*.[3] In input—output convention, by-products are treated as negative inputs of the process. The direct requirement coefficient matrix and environmental intervention coefficient matrix using the by-product-technology model are determined as follows:

$$A_B = (U - V'_{od})V_d^{-1} \tag{10}$$

and

$$E_B = BV_d^{-1}, \tag{11}$$

respectively, where V is split into V_d (diagonal entries in V) and V_{od} (off-diagonal entries in V). Subscript B denotes the by-product-technology model.

According to this method, the total direct and indirect environmental intervention, m_B, or LCI to produce a certain functional unit, k, is calculated by

$$m_B = E_B(I - A_B)^{-1}k \tag{12}$$

Mixed-Technology Model

One can apply more than one model to the same system by partitioning the system into several subsystems. This is relevant if different models are regarded as better reflecting the causal reality of a production process. For example, a refinery process that produces diesel, heavy oil, and gasoline may be treated with the industry-technology model, because production of each process output is not separable, and final demand of diesel or heavy oil can directly affect the operation schedule of the refinery process. If a process produces two heterogeneous products, however, such as an electroplating process that produces silver and nickel electroplating services, the commodity-technology assumption will better represent the input requirements and environmental intervention required to produce silver or nickel electroplating services. An allocation procedure in which more than one model is used for a system is referred to as a *mixed technology assumption* (see the work of Miller and Blair [1985] and Kop Jansen and Ten Raa [1990]).

Kop Jansen and Ten Raa (1990) discussed the choice of model using four axioms—namely, material balance, financial balance, scale invariance, and price invariance—and concluded that only the commodity-technology model successfully meets all the criteria (see also Ten Raa and Rueda-Cantuche 2003). The commodity-technology model is generally considered as the most theoretically desirable model, whereas the industry-technology model is considered as the most practical solution (Konijn 1994).[4] In practice, statistical offices often perform manual adjustments in addition to the use of these standard models. Note also that the commodity-technology and the by-product-technology models can result in negative values in the total requirement matrix. The reason for negative total requirements is simply that the total direct and indirect input requirements of a process to produce a certain amount of its primary product exceeds the total input requirements of a process that produces the same amount of the product as a nonprimary product. A negative sign will appear if the amount of product produced as a by-product is more than is directly and indirectly required to produce the primary product of the process. Thus, even though negative values seem unwarranted from a statistical viewpoint, they can be meaningful in an analytical sense, which means that production of a coproduct avoids production of inputs required to produce the same product as the primary product elsewhere (cf. Leontief 1970; Flick 1974; Leontief 1974; Steenge 1978; Lee 1982. See also the work of Hawkins and Simon [1949] for a nonnegativity condition for input–output systems and Suh and Heijungs [2007] for a generalization of the condition to LCA systems. In addition, see the Discussion section of this article).[5]

Identity Between By-product-Technology and Commodity-Technology Models

In this section, we argue that the distinction between the by-product-technology and commodity-technology models reviewed in the previous section does not have significant practical meaning when it comes to actual application of impact analyses and imputation studies. We acknowledge that these two methods have different underlying economic implications, which nevertheless does not have any effect in the results of an impact analysis or an imputation study.

The input–output literature has overlooked the fact that coefficient matrices (e.g., A and E) are rarely, if ever, used alone. Impact analysis and imputation studies use a coefficient matrix together with other information on primary factor use, such as labor input and environmental emissions. In such studies, one applies a certain demand to the coefficient matrix A to calculate the total output of products; one then applies this total output to a coefficient matrix of factor use, such as environmental burden (matrix E), after which one calculates the total factor use (matrix m). The coefficient matrices thus fulfill an intermediate function. Therefore, in this context these models can be better evaluated.

Let us consider a simple impact analysis or an imputation study that uses a product of primary input coefficient matrix and Leontief inverse, as shown in equations (9) and (12).

When we use equations (7) and (8), equation (9) becomes

$$m_{\mathrm{C}} = BV'^{-1}(I - UV'^{-1})^{-1}k \tag{13}$$

Likewise, using equations (10) and (11), equation (12) becomes

$$m_{\mathrm{B}} = BV_d^{-1}\left(I - (U - V'_{od})V_d^{-1}\right)^{-1} k \tag{14}$$

Equation (13) is then rearranged to[6]

$$\begin{aligned} m_{\mathrm{C}} &= B(V' - UV'^{-1}V')^{-1}k \\ &= B(V' - U)^{-1}k \end{aligned} \tag{15}$$

Likewise, equation (14) can be rearranged to

$$\begin{aligned} m_{\mathrm{B}} &= B\left(V'_d - \left(U - V'_{od}\right)V_d'^{-1}V'_d\right)^{-1} k \\ &= B\left(V'_d - U + V'_{od}\right)^{-1} k \\ &= B\left(V' - U\right)^{-1} k \end{aligned} \tag{16}$$

Therefore $m_{\mathrm{B}} = m_{\mathrm{C}}$ for any practical application in impact analyses or imputation studies.

This is not a coincidence, because these models are based on an equivalent underlying principle. The commodity-technology model isolates a unique input structure of a primary product of a multiproduct process by subtracting inputs required for coproducts by referring to the processes in which these products are produced as a single product. The by-product-technology model assumes that the coproducts produced by a multiproduct process substitute equivalent products produced by the processes in which these products are produced as a single product. The former is based on static reasoning, and the latter is based on dynamic reasoning. This subtle difference in underlying reasoning does not produce any difference when applied to impact analyses or to imputation studies, however.

Here we further argue that the two models discussed above—commodity-technology and by-product-technology models—are identical to the supply—use formulation of LCA proposed by Heijungs (1997, 2001) and Heijungs and Suh (2002). In LCAs, matrices are not necessarily converted into coefficient matrices,[7] and input and output quantities are directly used as they are. Heijungs and Suh (2002) proposed the use of the supply—use framework following the formula

$$m_{\mathrm{HS}} = B(V' - U)^{-1}k, \tag{17}$$

which appears at the end of the equations (15) and (16).[8]

Numerical Example

In this section, we discuss a fictitious numerical example to demonstrate the matrix operations presented in the previous sections for LCA applications. Suppose that the production and consumption of products by processes are noted by V' and U, respectively, as shown in figure 2 (note that V is presented in a transposed form in the figure). The example in figure 2 represents the three identified allocation situations: (1) multiple-output process, (2) recycling, and (3) multiple-input process. The multiple-output process is represented by the dairy farm process, which supplies both milk and cheese products. The recycling process is represented by copper recycling, which supplies both the disposal of waste copper as a service and the copper product. A multiple-input process is represented by the waste incineration process, which, like the recycling process, supplies the waste incineration service and electricity. In the V matrix, the diagonal entries represent the primary products of the activities, and off-diagonals represent by-products. For example, a dairy farm supplies 5 kg of cheese products in figure 2 as a by-product. In IOA, supply tables also include a column to the right representing imports. The total products output (row sum) is q, and the total process output (column sum) is g. Note that the column sum is possible because all of the products produced from each process are noted in a common unit. In IOA, there is a row representing payments to the primary inputs of the activities—that is, wages, taxes, and operating surplus (and sometimes also use of fixed capital)—below the U matrix, which is omitted in this example. Also excluded are the columns representing export and fixed capital formation. If the supply and use tables include all sectors and products in the economy and if they are in monetary units, the V and U tables can be balanced by sectors (g) as well as by

			Processes							
			Dairy farm	Cheese production	Copper mining	Power plant	Copper recycling	Waste incineration	Final use	Total (q)
V'	Products	Milk (kg)	100							100
		Cheese (kg)	5	80						85
		Copper (kg)			60		20			80
		Electricity ($)				30		5		35
		Disposal of waste copper (kg)					15			15
		Waste incineration service ($)						17		17
		Total (g')	105	80	60	30	35	22		
U	Products	Corn (kg)	2	50					48	
		Food (kg)							85	
		Copper (kg)	6	15	10	8	1	3	37	
		Electricity ($)	4	6	10		2		13	
		Disposal of waste copper (kg)	1	2	2				10	
		Waste incineration service ($)		3					14	
B	Emission	CO_2 (kg)	10	5	20	35	3	8		
		NO_x (kg)	2	1	6	8	1	2		

Figure 2 Illustration of life cycle inventory (LCI) data stored in the supply—use framework. The three types of allocation situations are illustrated: (1) multiple-output process, (2) recycling, and (3) multiple-input process. CO_2 = carbon dioxide; NO_x = nitrogen oxides.

products (q). If the tables are in physical units, this is also possible, but for g it requires that the primary inputs row contain inputs of resources and outputs (negative inputs) of emissions, wastes, and additions to stock.[9]

Suppose that the functional unit of a study is set as 100 kg of cheese. Applying the industry-technology model (equivalent to partitioning) on the basis of the example provided in figure 2 and using equations (4), (5), and (6), we have

$$A_{\mathrm{I}} = \begin{bmatrix} 0.019 & 0.589 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.057 & 0.18 & 0.13 & 0.25 & 0.029 & 0.14 \\ 0.038 & 0.073 & 0.14 & 0 & 0.057 & 0 \\ 0.0095 & 0.024 & 0.025 & 0 & 0 & 0 \\ 0 & 0.035 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$E_{\mathrm{I}} = \begin{bmatrix} 0.10 & 0.060 & 0.27 & 1.1 & 0.090 & 0.36 \\ 0.010 & 0.010 & 0.080 & 0.24 & 0.030 & 0.090 \end{bmatrix} \tag{19}$$

$$k = \begin{bmatrix} 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

$$m_{\mathrm{I}} = \begin{bmatrix} 36 \\ 8.8 \end{bmatrix} \tag{21}$$

Using the same functional unit but applying the by-product-technology model (equivalent to system expansion) on the basis of the example provided in figure 2 and using equations (10), (11), and (12), we have

$$A_{\mathrm{B}} = \begin{bmatrix} 0.02 & 0.63 & 0 & 0 & 0 & 0 \\ -0.050 & 0 & 0 & 0 & 0 & 0 \\ 0.060 & 0.19 & 0.17 & 0.27 & -1.3 & 0.18 \\ 0.040 & 0.075 & 0.17 & 0 & 0.13 & -0.29 \\ 0.010 & 0.025 & 0.033 & 0 & 0 & 0 \\ 0 & 0.038 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

$$E_{\mathrm{B}} = \begin{bmatrix} 0.10 & 0.060 & 0.33 & 1.2 & 0.20 & 0.47 \\ 0.010 & 0.010 & 0.10 & 0.27 & 0.067 & 0.12 \end{bmatrix} \tag{23}$$

$$m_{\mathrm{B}} = \begin{bmatrix} 39 \\ 8.6 \end{bmatrix} \tag{24}$$

Note that the resulting figures in equations (21) and (24) are close but not identical.^{10}

Generalization of the Allocation Situation

The allocation situation has been recognized in three generic cases: (1) the multiple-output process, (2) open loop recycling and (3) the multiple-input process (Lindfors et al. 1995). What follows is a generalization of the last two allocation situations as multiple-output processes.

Figure 3 illustrates a typical problem of open loop recycling. The product system under study is noted with the box with a dashed line. Discarded product A leaves the system under study and is used by another product system that produces product B. In this case, the system as a whole generates two functional outputs—the functional unit of the study and product B—and therefore inputs and outputs of the system need to be allocated over the two. The recycling process can be seen as a service input to either the supplying or the receiving process, depending on whether the recycling process is demanded by the supplier or the receiver of the discarded or recycled product A (see figures 4 and 5). Both system expansion and allocation take place at the point of substitution (the point at which the discarded or recycled product A can substitute a primary input to process B). This description is applicable both when the value of the discarded product is positive and when it is negative. In case the supply and use tables are in hybrid units, the service "to treat or recycle waste" can be represented by the amount of waste treated (in table *V*) and the amount sent to treatment (in table *U*) in physical units.

Likewise, one can consider the multiple-input process as a special case of multiple-output process. An incineration process, for instance, can be considered as a multiple-output process that provides, for example, disposal of tires, disposal of plastics, and generation of steam. In this case, provision of waste disposal service flows is in the opposite direction to the flow of waste.

In principle, either system expansion or the partitioning method can be applied to all allocation situations, including multi-input, multioutput, and open loop recycling cases, although in reality one method may be found more practical or more desirable than others. Therefore, all three allocation situations can be generalized as a multiple-output process, so all of the allocation

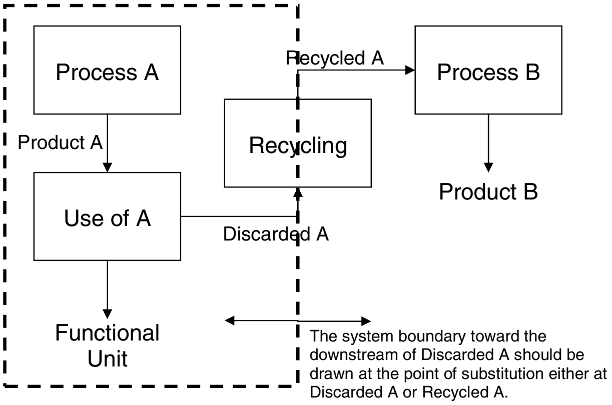


Figure 3 Open loop recycling as a multifunctional output problem.

situations can be represented with the generalized make and use framework described in the preceding sections. This also means that researchers can easily restructure input–output LCA databases to conform to the preferred allocation scheme used in process LCA or process LCA databases and vice versa (Suh 2005). The two data types therefore can be hybridized without the loss of methodological consistency.

Discussion

What follows are various topics that are relevant to the discussion so far.

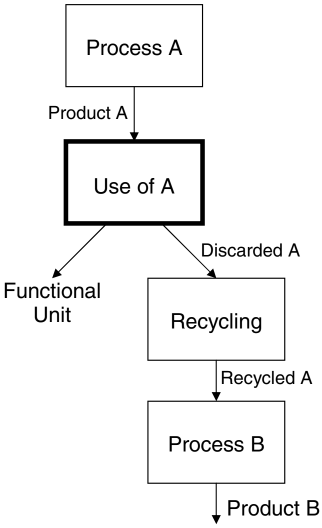


Figure 4 Open loop recycling considering Discarded A as a product.

Which Model to Choose?

The by-product-technology model shown in equation (17) is a preferred model whenever possible, for a couple of reasons. First, system expansion in general is a preferred solution both in LCA (ISO 1998) and in input–output economics (Jansen and ten Raa 1990; Konijn 1994). Second, unlike in the commodity-technology model, the coproduction structure is clearly and transparently shown in the by-product-technology model. According to the commodity-technology model, the technology matrix, A_C, is already converted into a single-product system, and thus the coproduction structure is not clearly visible. Third, the by-product-technology model is much simpler than the commodity-technology model and the industry-technology model.

It has been argued that in some cases this method alone cannot be used—namely, when

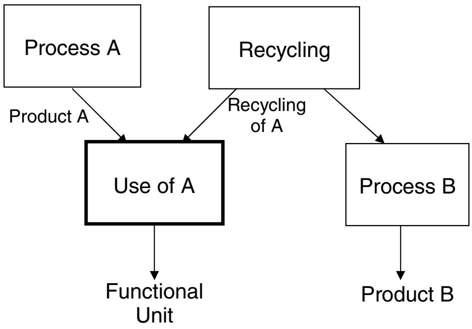


Figure 5 Recycling, with recycling services considered as a product.

there is no other process that produces its by-product as a main product. Examples are the production of copper and silver by a mining process, production of gasoline and diesel by a refinery, and production of different pork products from a pig. Weidema (2003) argued that in such cases either allocation can be avoided by a more detailed modeling of the necessary changes in the coproducing process or by inclusion of the necessary changes in consumption of the coproducts in the system expansion. One should bear in mind, however, that the by-product model, when applied to ready-made supply—use tables, leads to an automatic choice of the "avoided" process. In bottom-up LCA, a choice can be made if electricity coproduced through waste incineration substitutes coal-based electricity, nuclear electricity, or average electricity. As there is often just one entry for electricity in the supply—use tables, the choice is made implicitly, although one can easily "design" the product classification of supply and use tables to make a deliberate choice. Another problem in using the by-product-technology model is the possibility of negative solutions, which is discussed later in this section.

Relationship Between Attributional—Consequential Analyses in LCA and Impact—Imputation Study in IOA

In IOA, two types of analyses are distinguished: impact analysis, and imputation analysis. Impact assessment deals with the consequence of change in final demand or in investment in labor input, for instance. Questions such as "How many jobs can be created by a new construction project?" are typical impact analysis questions. Impact analysis therefore inherently deals with ex-ante questions. Imputation analysis, conversely, deals with content of factor inputs in final demand. The famous Leontief paradox (see the work of Duchin [1989] for an overview), for instance, was based on the observation that U.S. export was more labor intensive than capital intensive, which does not confirm the traditional theory of comparative advantage in international trade (Leontief 1953, 1956). Such questions are inherently ex-post questions. It is worth noting, however, that the distinction between ex-ante analysis and ex-post analysis is more general, and models of IOA are not limited to the change in final demand (see the work of Duchin and Steenge [2009] for different mathematical model types in IOA).

What is called impact analysis in input—output economics and the use of system expansion in LCAs can be considered in the context of consequential LCA. Impact analysis, system expansion, and consequential LCA all aim at modeling the impact of a specific change in the system. The mathematical formalism for the commodity-technology and by-product-technology models is equivalent to a 1:1 substitution of characteristic products by coproducts (cf. Weidema 2003).

Imputation analysis in input—output economics is analogous to attributional LCA. In an imputation—attributional study, one can use both the industry-technology model and the commodity-technology model for allocation without contradicting the model's underlying reasoning. Recall that a fully static reasoning is possible for the commodity-technology model, whereas the results will be the same as those from consequential reasoning under certain conditions.

Relationship Between System Expansion and the By-product-Technology Model

Weidema (2001) showed how system expansion can be used under various conditions. As previously discussed, the commodity-technology model and the by-product-technology model work from slightly different underlying reasoning from an input-output economics viewpoint, whereas the two produce the same results in practical impact analyses and imputation studies: The commodity-technology model can be understood as a procedure to isolate a unique input structure for each product by subtracting inputs for coproducts in reference to the process in which the coproducts are produced alone, whereas the by-product-technology model can be understood as a substitution mechanism, in which by-products as a price-taker replace stand-alone production. In this sense, the by-product-technology model is closer to the reasoning behind the consequential allocation of Weidema (2001). In fact, the by-product-technology model follows the same reasoning as the proposal by Weidema (2001) under the following conditions: (1) the prices of by-products are adjusted so that the market is always cleared, (2) there is a 1:1 direct substitution by by-products that takes into account price elasticity and substitution capacity, and (3) the amount of by-product produced by the system is less than the amount of total demand by the economy on the equivalent product. The first two conditions define the substitution mechanism, and the third prevents substitution beyond the market capacity. In the case of joint production, the system expansion method by Weidema (2001) is reduced to the partitioning method.

Relationship With Other Models

Nakamura and Kondo (2002) laid out the basic framework of the waste input—output (WIO) model, which has been extended to embrace various areas, such as life cycle costing and material contents problems (Kondo and Nakamura 2004; Nakamura and Nakajima 2005). One of the problems that motivated the development of the WIO model was the fact that multiple types of wastes are treated by a waste treatment process, which leads to a rectangularity problem when an input—output framework is utilized to analyze them (see Nakamura 1999). In this model, the flows of waste and scrap generated by industries are specified in physical units, and waste treatment methods, such as landfill and incineration, are disaggregated in an input—output table. One of the unique features of this model is the "allocation matrix" (see, e.g., Nakamura and Kondo 2002, 44). The allocation matrix is introduced in the WIO model because the number of waste and scrap types is larger than the number of waste treatment methods, which leads to a rectangular technology matrix, which is not invertible. The allocation matrix in the WIO model is in the dimension of waste treatment method-by-waste types and is premultiplied to the rows that describe waste flows (see also Kondo and Nakamura 2004). Although its mathematical formalism looks different, the allocation matrix used in the WIO model can be interpreted according to the standard supply and use framework discussed in the current article. The allocation matrix in the WIO model can be viewed as what is generally referred to as *market share matrix* in supply and use literature (Konijn 1994). In equation (4) of this article, we postmultiply the normalized use matrix by the market share matrix to derive the commodity-by-commodity technology matrix. Instead, one can interpret the allocation step in the WIO model as a process in which the normalized use matrix is premultiplied by the market share matrix, which is a standard procedure used to derive an industry-by-industry (in this case, waste treatment method-by-waste treatment method) technology matrix using industry-technology model (see, e.g., Horowitz and Planting 2006, Chapter 12). Unlike the standard industry-technology model, however, the WIO model uses both positive and negative signs for the rows that describe waste flows, so it can accommodate the use of assumptions similar to those of the byproduct-technology model as well as those of the industry-technology model, although, strictly speaking, the byproduct-technology model is generally not applicable in an industry-by-industry setting.

Hybrid LCA models integrate detailed process analysis with an input—output table (Treloar 1997; Joshi 1999; Matthews and Small 2001; Lenzen 2002; Suh 2004a; Suh et al. 2004). Some of these models, such those by as Joshi (1999) and Suh (2004a), combine process analysis and input—output tables in a single, concatenated matrix using a consistent mathematical principle (see the work of Suh and Huppes [2005] for a review). The supply and use framework discussed in this article can be applied to these models without loss of generality.

Negative Values in the Results

Almon (2000) provides a seminal review on the negative elements in the inverse of a coefficient matrix generated from either the by-product-technology or the commodity-technology model. Konijn (1994) lists three reasons why negatives occur. First, there is more than one way to produce coproducts. In this case, the process that produces the coproduct in question may require fewer factor inputs than the coproduction process; therefore, substitution or subtraction results in negative values.

Second, heterogeneous products are aggregated into one commodity, which disturbs the substitution or subtraction procedure. Third, data error in supply—use matrices may also lead to negatives in the results. Through the use of detailed, process-specific data, some of these causes of negative values can be prevented in LCA. In general, negative values in the result are better tolerated in LCA than in IOA. In LCA, negative values are interpreted as a "credit" or "avoidance" of environmental emissions or natural resources extraction. Negatives in the results of an impact analysis or of a consequential analysis should be treated with a caution, however, because negatives may occur by *oversubstitution*. Oversubstitution is when the amount of coproducts produced exceeds the amount demanded by the system under study, so that the by-product-technology and commodity-technology models grant credits to the coproduction process beyond what can be possibly substituted. In this case, the excess by-product should be treated as a waste, because the by-product will not be used in the market for productive purposes (see also Weidema 2001). In this case, the results from the by-product-technology model with alternative treatment to the excess by-product deviate from the results from the commodity-technology model.

The Original Literature That Coined the Relation $m = B(V' - U)^{-1}k$

In the *Identity Between By-product-Technology and Commodity-Technology Models* section, we argued that the commodity-technology model, the by-product-technology model, and the approach proposed by the LCA community are equivalent when they are used for practical applications of impact analysis, imputation studies, and LCA studies. In that section, we derived the simple formula $m = B(V' - U)^{-1}k$ from the commodity-technology model and the by-product-technology model, proving the equivalence of all the three approaches. To our best knowledge, even the equivalence between the commodity-technology model and the by-product-technology model has remained unnoticed over the last half-century since Richard Stone (1961) developed the two models in the early 1960s. Throughout the literature, the two models have been analyzed as two different approaches, due mainly to the emphasis placed on coefficient tables in those analyses (see, e.g., van Rijckeghem 1967; Kop Jansen and Ten Raa 1990; Ten Raa and Rueda-Cantuche 2003).

Nevertheless, tracing back the original literature that coined the relation, $m = B(V' - U)^{-1}k$, would be of interest.[11] It is interesting that the technique of noting production and consumption separately in a multiproduct system, which can be translated to the use of V and U matrices in the supply—use tradition, was already in practice even before the work of Stone (1961;(see, e.g., von Neuman 1945–1946; Sraffa 1960). Efforts to link such early works with the supply—use framework started much later, however. [12] To our best knowledge, Ten Raa and Wolff (1994, 11, equation 11) first derived an equivalent equation to $m = B(V' - U)^{-1}k$ from the commodity-technology model. The derivation appears in a research report by the authors, although a peer-reviewed publication based on the report (ten Raa and Wolff 2001) somehow omitted the derivation. The first peer-reviewed article that derived the relation was by Bidard and Erreygers (1998, 436, equation 20), who also derived the relation from the commodity-technology model. In particular, Bidard and Erreygers (1998) related the formula as a common ground that connects Sraffian and input—output traditions. Nonetheless, the relationship between the formula and the by-product-technology model was not discussed in these early works. From the LCA side, Heijungs (1997, 70, equation 6.10) first coined the equation in his Ph.D. thesis, in which he discusses a possible link between the supply—use framework and LCA. These early derivations of the equation seem to have stemmed from independent research, given from the differences in motivation and the references cited in these works. Later, the equation reappears in the work by Heijungs and Suh (2002) and Ten Raa and Rueda-Cantuche (2007), among others.

Conclusions

In this article, we have presented a coherent mathematical framework for allocation in LCA using the supply—use framework of input—output economics. We have shown that the partitioning method is equivalent to the industry-technology model and that system expansion is equivalent to the by-product-technology model. We have also shown that the commodity-technology model and the by-product-technology model, which have been treated as two different models since they first appeared in the input–output literature in the 1960s, are, in fact, equivalent when applied to practical applications.[13] In addition, we have shown that the supply–use formula for LCA coined by Heijungs (1997) and Heijungs and Suh (2002) is equivalent to the commodity-technology model and thus to the by-product-technology model in practical applications. To the best of our knowledge, the practical identity between these seemingly disparate frameworks across LCA and IOA—namely, the commodity-technology model, the by-product-technology model, and the supply-use formula by Heijungs (1997, 2001) and Heijungs and Suh (2002), has not been recognized before.

We presented a numerical example to demonstrate the usefulness of the approaches. Finally, we showed that all three types of allocation situations in LCA—namely, (1) multiple-output process, (2) open loop recycling, and (3) multiple-input process—are generalized and thus that all allocation situations can be represented with the make and use framework.

The implication of the findings for LCA is that allocation of even very large-scale LCI problems, such as those in commercial LCI databases, can be computed with a consistent mathematical framework, which can avoid numerous hardly traceable manual handlings. This also makes it easier to switch among different allocation methods for, for example, sensitivity analysis. The generalized make and use framework can also handle a combination of allocation methods by using the so-called mixed-technology model (see, e.g., Miller and Blair 1985). The framework described in this article can easily be implemented in widely used LCA software tools.

Acknowledgement

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Notes

1. Some early literature also use *substitution method* or *avoided impact method* for what is equivalent to *system expansion* (see, e.g., Huppes and Schneider 1994; Lindfors et al. 1995). Tillman and colleagues (1994) demonstrate this equivalence, but, as noted by Heijungs and Guinée (2007), equivalence does not mean identicalness. In fact, there is a subtle difference between (1) expanding the system and including additional functions (ISO 1998, 11) and (2) subtracting avoided processes while maintaining the original function (ISO 1998, 20). In the remainder of this article, we focus on the substitution–avoided burdens method, but we often refer to it as a system expansion.
2. Such an assumption is a strong one, and the number of products and the number of processes of a system can be different, which is often referred to as a *rectangularity problem* (Konijn 1994). The assumption does not, however, constrain generalization of the results when the industry-technology model is used alone or in combination with others. If either the commodity-technology model or the by-product-technology model is to be used alone, relevant reclassification is needed to make a square matrix, which is necessary for a matrix inversion.
3. This method is often called "Stone's method" (Stone 1961; Konijn 1994).
4. This perception on supply–use models in input–output economics resembles that of the LCA community on allocation methods, where system expansion is the preferred solution (ISO 1998) and economic allocation is perhaps more popular.
5. A process or a facility that produces multiple products as coproducts or by-products may use less input per unit of each output than a process or facility that produces each as a stand-alone product, because the multiproduct system may share some common factor inputs across the outputs. In this case, subtracting total input requirements to produce a stand-alone product from requirements to produce the same product as a by-product also leaves negatives in the total requirement matrix.
6. The identity $A^{-1}B^{-1} = (BA)^{-1}$ is used when these equations are rearranged.
7. That is to say, there is no strict need, and the formulas work neatly without coefficients. Nevertheless, it is quite usual to do so. ISO 14041 (ISO

1998, 10) discusses "relating data to unit process," and a database like Ecoinvent contains, in fact, coefficient tables. But we get exactly the same results when we skip this normalization to a unit output.

8. See the Discussion section of this article for the original literature that coined the formula.
9. See the work of Hubacek and Giljum (2003), Suh (2004b), Weisz and Duchin (2006), and Dietzenbacher and colleagues (2009) for discussions on balancing input—output tables in physical units.
10. The numerical example is for illustration purposes only and does not represent real-life cases.
11. This issue was raised by one of the referees. We note that, given the large amount of literature on the subject spanning the last half-century, our analysis might have missed some of the earlier works. Readers are invited to share their knowledge in this regard if we have omitted any important prior works.
12. Despite the important commonalities in the underlying accounting frameworks among the pioneering works by von Neuman (1945–1946), Sraffa (1960), and Stone (1961), attempts to formally link the neoclassical and Sraffian roots with the supply—use framework were made only relatively recently, by, for example, Ten Raa and Mohnen (1994) and Bidard and Erreygers (1998). See also the work of Dorfman and colleagues (1958), an early work that discusses IOA in transition to neoclassical models.
13. Nevertheless, one should not ignore the difference of the two models as theoretical constructs to extract a unique input structure of a product.

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About the Authors

Sangwon Suh is a professor in Bren School of Environmental Science and Management at the University of California in Santa Barbara, California, USA. **Bo P. Weidema** is an external associate professor and **Jannick Hoejrup Schmidt** is an assistant professor, both in the Department of Development and Planning at Aalborg University in Aalborg, Denmark. **Reinout Heijungs** is an assistant professor at the Institute of Environmental Sciences (CML) at Leiden University in Leiden, the Netherlands.